Radiation effect on Non-Darcy Convective Heat Transfer through a porous medium in a vertical channel with heat generating source

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Abstract : In this study the effect of radiation on the Non-Darcy convection heat transfer flow of a viscous electrically conducting fluid through a porous medium confined in a vertical channel investigated by taking into account both heat generating source and Radiation effect in the presence of heat sources. The non-linear, coupled equations governing the flow and heat transfer have been solved by employing a perturbation technique with δ the porous parameter as perturbation parameter. The velocity and temperature dissipation are analysed for different values of G,D^{-1}, α , and M .From the analysis, new expressions for Nusselt number and Shear Stress on the walls are numerically evaluated for different sets of parameters

Keywords: Convective Heat transfer, CFD, Porous medium, Vertical Channel, Radiation effect

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1. Introduction

Any substance with a temperature above zero transfers heat in the form of radiation. Thermal radiation always exits and can strongly interact with convection in many situations of engineering interest. The influence of radiation on natural or mixed convection is generally stronger than that on forced convection because of the inherent coupling between temperature and flow field [1]. Convection in a channel (or enclosed space) in the presence of thermal radiation continues to receive considerable attention because of its importance in many practical applications such as furnaces, combustion chambers, cooling towers, rocket engines and solar collectors. During the past several decades, a number of experiments and numerical computations have been presented for describing the phenomenon of natural (or mixed) convection in channels or enclosures.

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These studies aimed at clarifying the effect of mixed convection on flow and temperature regimes arising from variations in the shape of the channel (or enclosure),in fluid properties, in the transition to turbulence, etc.
Akiyama and chang [2] numerically analysed the influence of gray surface radiation on the convection of nonparticipating fluid in a rectangular enclosed space.

Barletta and Magyari[3] discussed the convention with viscous dissipation in the thermal entrance region of a circular duct.Numerous studies have been denoted to the investigation of thermal stability of reactors with internal volumetric heat generation and cooling of the walls. For example, In classical monographs [4,5] ,the emergence of uncontrolled rise of temperature in a reactor with internal heat release is referred to as thermal explosion.

Recently, Ghaly [6], Chamkha et al.[7], Yussyo El-Dib and Ghaly[8], analyze the effect of radiation heat transfer on flow and thermal fields in the presence of a magnetic field for horizontal and inclined plates. Shohel Mahmud [9] studied the effects of radiation heat transfer on magneto hydrodynamic mixed convection through a vertical channel packed with fluid saturated porous substances.

Farajollahi et al [10] published studies on the forced convective heat transfer coefficient of nanofluids and most of them are under the constant heat flux or constant temperature boundary conditions at wall of tubes and channels.

The early research which used suspension and dispersion of millimeter-and micrometer-sized particles, faced the major problem of poor suspension stability.Thus, a new class of fluid for improving both thermal conductivity and suspension stability is required in the various industrial fields.This motivation leads to development of nanofluids [11] Kyo Sik Hwang

Non-Darcy effects on natural convection in porous media have received a great deal of attention in recent years because of the experiments conducted with several combinations of solids and fluids covering wide ranges of governing parameters which indicate that that the experimental data for systems other than glass water at low Rayleigh members do not agree with theoretical predictions based on the Darcy flow model. This divergence in the heat transfer results has been reviewed in detail in Cheng [12] and Prasad et al., [13] among others.

Haddad and Abuzaid [14] the developing free convection flow in a vertical open-ended micro-channel with porous media. Raveendra Nath et al [15] applied the computation for the analysis convective heat and mass transfer through a porous medium with heat sources and dissipation.

Rokini and Sunder [16] developed computational method for straight ducts with arbitary cross-sections to predict turbulent Reynolds stresses and turbulent heat fluxes in ducts by different turbulence models for fully **developed conditions.**

2. The problem formulation

We consider a fully developed laminar convective heat transfer flow of a viscous, electrically conducting fluid through a porous medium confined in a vertical channel bounded by flat walls. We choose a Cartesian co-ordinate system O(x,y,z) with x- axis in the vertical direction and y-axis normal to the walls. The walls are taken at y=$\pm$1.The walls are maintained at constant temperature and concentration. The temperature gradient in the flow field is sufficient to cause natural convection in the flow field .A constant axial pressure gradient is also imposed so that this resultant flow is a mixed convection flow. The porous medium is assumed to be isotropic and homogeneous with constant porosity and effective thermal diffusivity. The thermo physical properties of porous matrix are also assumed to be constant and Boussinesq approximation is invoked by confining the density variation to the buoyancy term. In the absence of any extraneous force flow is unidirectional along the x-axis which is assumed to be infinite.

The Brinkman-Forchheimer-extended Darcy equation which account for boundary inertia effects in the momentum equation is used to obtain the velocity field. Based on the above assumptions the governing equations in the vector form are

Equation of Continuity

$$\nabla.\bar{q} = 0 \quad (1)$$

Equation of linear momentum

$$\frac{\rho}{\delta}\frac{\partial \bar{q}}{\partial t} + \frac{\rho}{\delta^2}(\bar{q}.\nabla)\bar{q} = -\nabla p + \rho g - \left(\frac{\mu}{k}\right)\bar{q} - \left(\frac{\rho F}{\sqrt{k}}\right)\bar{q}.\bar{q}. + \mu\nabla^2\bar{q} + \mu_e(\bar{J}x\bar{H}) \quad (2)$$

Equation of energy

$$\rho C_p\left(\frac{\partial T}{\partial t} + (\bar{q}.\nabla)T\right) = \lambda\nabla^2 T + Q(T_o - T) - \frac{\partial(q_r)}{\partial y} \quad (3)$$

Equation of State

$$\rho - \rho_0 = -\beta\,\rho_0(T - T_0) \quad (4)$$

Since the flow is unidirectional, the continuity of equation (1) reduces to $\frac{\partial u}{\partial x} = 0$ Where u is the axial velocity implies u = u(y)

The momentum, energy and diffusion equations in the scalar form reduces to

$$-\frac{\partial p}{\partial x} + \left(\frac{\mu}{\delta}\right)\frac{\partial^2 u}{\partial y^2} - \left(\frac{\mu}{k}\right)u - \frac{\rho\delta F}{\sqrt{k}}u^2 - \left(\frac{\sigma\mu_e^2 H_o^2}{\rho_0}\right)u - \rho g = 0 \quad (5)$$

$$\rho_0 C_p u\frac{\partial T}{\partial x} = \lambda\frac{\partial^2 T}{\partial y^2} + Q(T_o - T) - \frac{\partial(q_r)}{\partial y} \quad (6)$$

The boundary conditions are

$$u = 0\ ,\ \ T = T_1\ on\ \ y = -L \quad (7)$$

$$u = 0\ ,\ \ T = T_2\ on\ \ y = +L \quad (8)$$

The axial temperature gradients $\frac{\partial T}{\partial x}$ is assumed to be a constant, say, A .Invoking Rosseland approximation for radiative heat flux

$$q_r = -\frac{4\sigma^\bullet}{\beta_R}\left(\frac{\partial(T'^4)}{\partial y}\right)$$

and expanding T'^4 about Te by Taylor's series and neglecting the higher order terms we get

$$T'^4 \cong 4T_e^3 T' - 3T_e^4$$

where $\sigma^\bullet$ is the Stefan-Boltzman constant and β_R is mean absorption coefficient.

We define the following non-dimensional variables as

$$u' = \frac{u}{\left(\frac{\nu}{L}\right)},\ (x', y') = \frac{(x, y)}{L},\ \ p' = \frac{p\delta}{\left(\frac{\rho\nu^2}{L^2}\right)}, \theta = \frac{T - T_2}{T_1 - T_2} \quad (9)$$

Introducing these non-dimensional variables the governing equations in the dimensionless form reduce to (on dropping the dashes)

$$\frac{d^2u}{dy^2} = \pi + \delta\left(D^{-1} + M^2\right)u - \delta G(\theta + NC) \quad (10)$$

$$\frac{d^2\theta}{dy^2} - \alpha\,\theta = (PN_T)u \quad (11)$$

where

$A = FD^{-1/2}$ (Inertia or Fochhemeir parameter)

$$G = \frac{\beta g(T_1 - T_2)L^3}{\nu^2}$$ (Grashof Number)

$$D^{-1} = \frac{L^2}{k}$$ (Darcy parameter)

$$P = \frac{\mu C_p}{\lambda}$$ (Prandtl Number)

$$\alpha = \frac{QL^2}{\lambda}$$ (Heat source parameter)

The corresponding boundary conditions are

$$u = 0 \ , \ \theta = 1 \quad on \ y = -1 \quad (12)$$

$$u = 0 \ , \ \theta = m \quad on \ y = +1 \quad (13)$$

3.Shear Stress and Nusselt number

The shear stress on the boundaries $y = \pm 1$ is given by

$$\tau_{y=\pm L} = \mu\left(\frac{du}{dy}\right)_{y=\pm L} \quad (14)$$

which in the non-dimensional form is

$$\tau_{y=\pm 1} = \left(\frac{du}{dy}\right)_{y=\pm 1} \quad (15)$$

and the corresponding expressions are

$$\tau_{y=+1} = \pi + \delta b_{31} + \delta^2 b_{37} \quad (16)$$

$$\tau_{y=-1} = \pi + \delta b_{32} + \delta^2 b_{38} \quad (17)$$

The rate of heat transfer (Nusselt Number) is given by

$$Nu_{y=\pm i} = \left(\frac{d\theta}{dy}\right)_{y=\pm 1} \quad (18)$$

and corresponding expressions are

$$Nu_{y=+1} = b_{27} + \delta b_{33} + \delta^2 b_{39} \quad (19)$$

$$Nu_{y=-1} = b_{28} + \delta b_{34} + \delta^2 b_{40} \quad (20)$$

4. Results and discussions

In this analysis we discuss the velocity, temperature, shear stress and rate of heat transfer with the variation in different governing parameters viz., Grashof number G, Darcy parameter D^{-1}, heat source parameter α , and Hartmann number M .The governing equations of momentum and energy are solved by perturbation method with δ as a perturbation parameter and we take P=0.71. The velocity is exhibited in Figures (1-4) for different G, D^{-1}, α , and M .

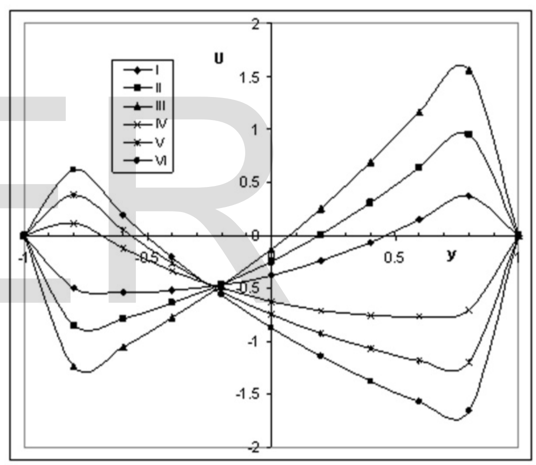

Fig.1

Fig(1) Variation of u with G

N_1=0.5,M=2,D^{-1}=10^2 G=10^2, α =2

	I	II	III	IV	V	VI
G	10^2	$2x10^2$	$3x10^2$	-10^2	$-2x10^2$	$-3x10^2$

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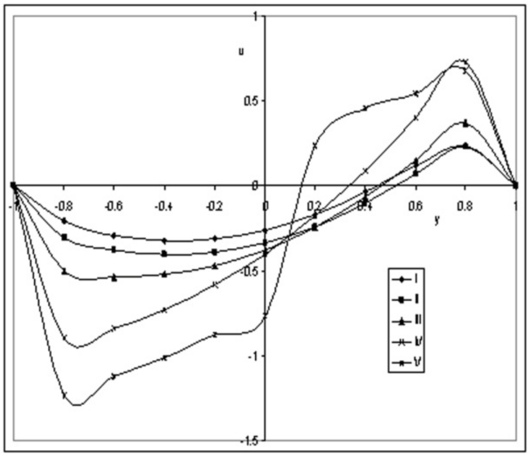

Fig.2

Fig(2) Variation of u with D^{-1}

$N_1=0.5, M=2, G=10^2, \alpha=2$

	I	II	III	IV	V	VI
D	10^2	10^2	10^2	10^2	$3x10^2$	$5x10^2$

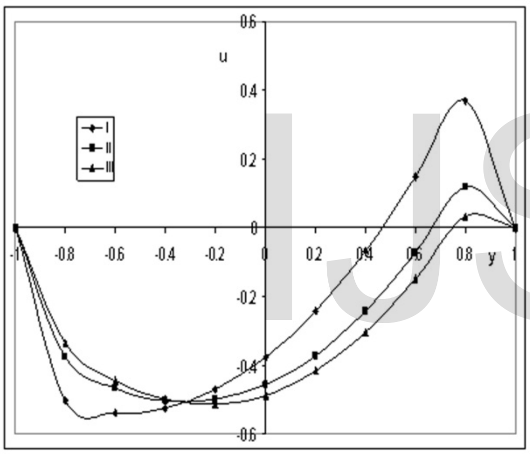

Fig 3

Fig(3) Variation of u with α

$N_1=0.5, M=2, D^{-1}=10^2\ G=10^2$

	I	II	III
α	2	4	6

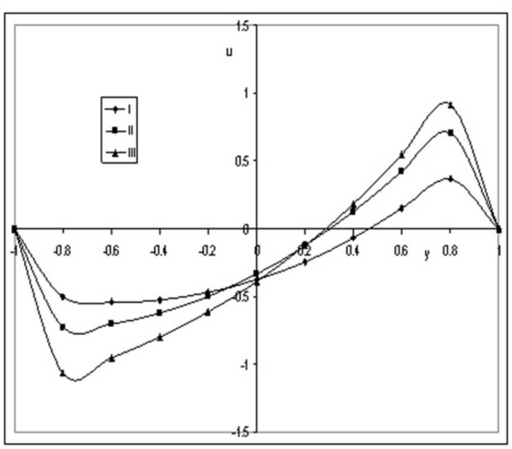

Fig.4

Fig(4) Variation of u with M

$N_1=0.5, D^{-1}=10^2, G=10^2, \alpha=2$

	I	II	III
M	2	4	6

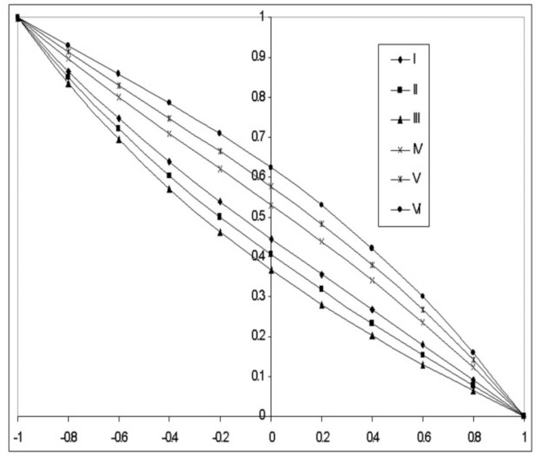

Fig.5

Fig(5) Variation of θ with G

$N_1=0.5, M=2, D^{-1}=10^2\ G=10^2, \alpha=2$

	I	II	III	IV	V	VI
G	10^2	$2x10^2$	$3x10^2$	-10^2	$-2x10^2$	$-3x10^2$

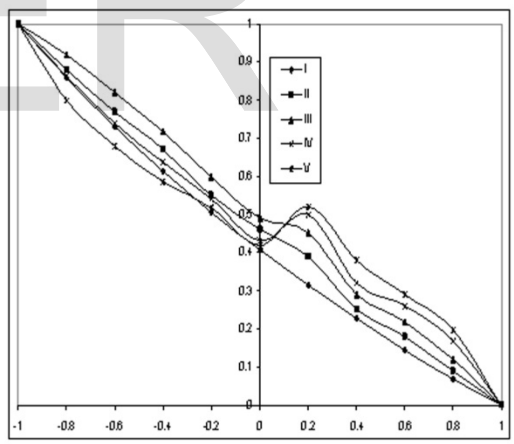

Fig.6

Fig(6) Variation of θ with D^{-1}

$N_1=0.5, M=2, G=10^2, \alpha=2$

	I	II	III	IV	V	VI
D^{-1}	10^2	10^2	10^2	10^2	$3x10^2$	$5x10^2$

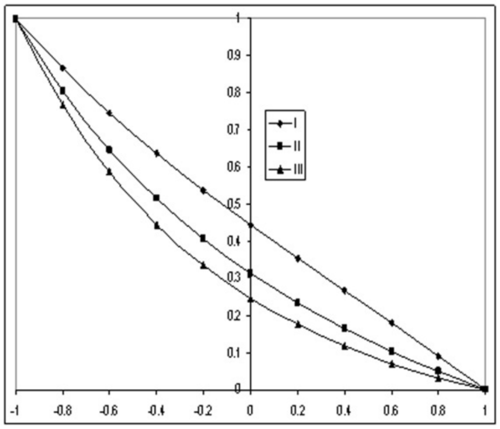

Fig.7

Fig(7) Variation of θ with α

N_1=0.5,M=2,D^{-1}=10^2 G=10^2

	I	II	III
α	2	4	6

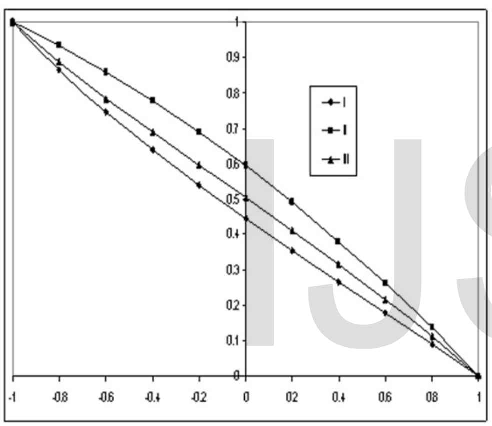

Fig.8

Fig(8) Variation of θ with M

N_1=0.5,D^{-1}=10^2, G=10^2, α =2

	I	II	III
M	2	4	6

From Fig.1 it is found that as we move from the left boundary to right boundary the velocity changes from negative to positive. The region of transition from negative to positive near the boundary enlarges with increase in G (>o). For variation in |G| (<0) this transition takes place near the boundary and the transition region increases with |G|. For G> 0 the maximum u occurs at y = -0.6 and for higher G$\geq$2x10^3 the maximum occurs in the vicinity of the right boundary.

The variation of u with D^{-1} is shown in Fig.2. We notice that the region of transition from negative to positive shrinks in its size with increase in $D^{-1} \leq 10^3$ and for higher D^{-1} it enlarges. In this case the maximum u occurs in the vicinity of the lower boundary. Also lesser the permeability of the porous medium larger the magnitude of u.

Fig.3 shows that the magnitude of u decreases with the increase in the strength of the heat generating source α. Also the region of transition of velocity near the right boundary shrinks with increase in α.

With reference to M the transition region enlarges with increase in M. Also |u| experiences an enhancement with M (fig.4).

The non-dimension temperature (θ) is shown in the Figs.(5-8) for different G, D^{-1}, α, and M . For all variations the temperature is positive in the entire flow region. This means that the actual temperature is greater than the equilibrium temperature in the fluid region. The temperature gradually reduces from the value 1 on the left boundary to attain its prescribed value 0 on the right boundary y=1. It is found that the temperature depreciates with G>0 and enhances with increase in |G|(<0)(Fig.5).

The variation of θ with D^{-1} exhibits that lesser the permeability of porous medium larger the temperature in the flow region (Fig.6).

The variation of θ with α shows that θ depreciates with increase in the strength of the heat generating source α (Fig.7)

The variation of θ with M implies that θ experiences an enhancement with M$\leq$4 and for higher M$\geq$ 6 it experiences an enhancement in the flow region (Fig.8).

Table. 1Shear Stress(τ) at y=1P=0.71,N_1=4.0

G	I	II	III	IV	V	VI	VII
10^3	1.16817	1.18517	1.3382	1.17899	1.22454	1.04591	1.00704
2x10^3	1.28032	1.34675	1.66583	1.33158	1.43287	1.08008	1.01034
3x10^3	1.3363	1.48467	1.98285	1.45769	1.62495	1.10236	1.00976
-10^3	0.77539	0.791	0.65108	0.79442	0.75913	0.94193	0.98882
-2x10^3	0.49475	0.55841	0.2916	0.56242	0.50205	0.87211	0.97389
-3x10^3	0.15795	0.30216	-0.07849	0.30396	0.22872	0.7904	0.9551

M	2	4	6	2	2	2	2
D^{-1}	10^3	10^3	10^3	2x10^3	3x10^3	10^3	10^3
α	2	2	2	2	2	4	6

Table. 2Shear Stress(τ) at y=-1P=0.71,N_1=4.0

G	I	II	III	IV	V	VI	VII
10^3	-1.1466	-1.06243	-0.85743	-1.0767	-0.99751	-1.0567	-1.02828
$2x10^3$	-1.25562	-1.10793	-0.7071	-1.13455	-0.98321	-1.11028	-1.05808
$3x10^3$	-1.32693	-1.13641	-0.54896	-1.17347	-0.95707	-1.1606	-1.08935
-10^3	-0.81541	-0.9204	-1.13465	-0.90425	-0.99057	-0.93977	-0.97292
$-2x10^3$	-0.59325	-0.82387	-1.26154	-0.78964	-0.96934	-0.87643	-0.94742
$-3x10^3$	-0.33337	-0.71032	-1.38062	-0.65611	-0.93626	-0.80984	-0.92337

M	2	4	6	2	2	2	2
D^{-1}	10^3	10^3	10^3	$2x10^3$	$3x10^3$	10^3	10^3
α	2	2	2	2	2	4	6

Table .3Nusselt Number(Nu) at y=1P=0.71,N_1=0.4

G	I	II	III	IV	V	VI	VII
10^3	-4.69955	-4.74269	-4.76758	-4.73867	-4.7547	-2.94674	-2.29748
$2x10^3$	-4.53249	-4.66412	-4.73637	-4.65174	-4.70027	-2.88639	-2.26177
$3X10^3$	-4.38299	-4.59495	-4.70993	-4.5751	-4.65272	-2.83307	-2.23032
-10^3	-5.08633	-4.92807	-4.84426	-4.94341	-4.88419	-3.08853	-2.37987
$-2x10^3$	-5.30605	-5.03486	-4.88973	-5.06121	-4.95927	-3.16998	-2.42716
$-3x10^3$	-5.54333	-5.15107	-4.93996	-5.18931	-5.04122	-3.25845	-2.47841

M	2	4	6	2	2	2	2
D^{-1}	10^3	10^3	10^3	$2x10^3$	$3x10^3$	10^3	10^3
α	2	2	2	2	2	4	6

Table. 4Nusselt Number(Nu) at y=-1P=0.71,N1=0.4

G	I	II	III	IV	V	VI	VII
10^3	4.7	4.729	4.735	4.76	4.734	2.942	2.292
$2x10^3$	4.557	4.643	4.673	4.635	4.663	2.883	2.253
$3x10^3$	4.455	4.573	4.618	4.562	4.602	2.837	2.221
-10^3	5.11	4.948	4.879	4.963	4.909	3.099	2.388
$-2x10^3$	5.377	5.082	4.961	5.109	5.012	3.198	2.445
$-3x10^3$	5.685	5.232	5.051	5.272	5.126	3.309	2.509

M	2	4	6	2	2	2	2
D^{-1}	10^3	10^3	10^3	$2x10^3$	$3x10^3$	10^3	10^3
α	2	2	2	2	2	4	6

The shear stress and the rate of heat transfer at the boundaries is exhibited in the tables 1 and 2 for different sets of the governing parameters G, D^{-1}, α , and M . It is found that the shear stress at the right boundary $y=1$ is positive for all variations. The shear stress increases in the heating case and reduces in the cooling case. It enhances with M$\leq$4 and depreciates for higher $M \geq 6$ at G = 10^3. For higher G$\geq 2x10^3$ the shear stress enhances with M. In the case of cooling |G|(<0) we notice that the shear stress enhances with M$\leq$4 and reduces with M$\geq$6. Also in case of cooling, the stress enhances with increase in D^{-1} in the heating of the channel while in the cooling case the shear stress increase with $D^{-1} \leq 2x10^3$ and reduces with $D^{-1} \geq 3x10^3$. Also stress experiences a depreciation with increase in the strength of the heat generating source α (Table1).

Table.2 shows that The shear stress at left boundary is negative for all variations. The magnitude of stress increases with G>0 and reduces with G<0. It is found that |τ| depreciates with M in the heating case and enhances in the cooling case. Also lesser the permeability of porous medium smaller |τ| for G>0 and larger |τ| for G <0. With reference to α, shear stress depreciates with α in heating case and enhances with α in the cooling case.

The Nusselt number (Nu) which measures the rate of heat transfer across the boundaries is exhibited in the tables 3 and 4 for different sets of variation in G, D^{-1}, M, and α . The rate of heat transfer at y=1 is negative for all variations and that at y=-1 it is positive for all variations. It is found that |Nu| reduces with G>0 and enhances with |G|(<0).Also |Nu| at both the boundaries enhances with M in the heating case and reduces with it in the cooling case. The variation of Nu with D^{-1} shows that lesser the permeability of porous medium larger |Nu| at y= ± 1 in the heating case and smaller |Nu| in the cooling case. Also |Nu| experiences a depreciation at y= ± 1 with increase in α for all G (Tables 3 and 4).

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Nomenclature

$\bar{q}$ velocity(m/s)
P Pressure(N/m^2)
k Permeability of the porous medium (H/m)
Q Strength of the heat generating source
C_p Specific heat at constant pressure(J/kg K)
$\bar{J}$ Current density vector
H The magnetic field vector
T Temperature in the flow region(K)

Greek Symbols

ρ Density of the fluid (Kg/m^3)
μ Coefficient of viscosity(Ns/m^2)
λ Coefficient of thermal conductivity(W/mk)
β Coefficient of thermal expansion(m^3)
δ Porosity of the medium